

Possible Measurable Effects of Dark Energy in Rotating Superconductors

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February 1, 2008

Abstract

We discuss recent laboratory experiments with rotating superconductors and show that three so far unexplained experimentally observed effects (anomalous acceleration signals, anomalous gyroscope signals, Cooper pair mass excess) can be physically explained in terms of a possible interaction of dark energy with Cooper pairs. Our approach is based on a Ginzburg-Landau-like model of electromagnetic dark energy, where gravitationally active photons obtain mass in the superconductor. We show that this model can account simultaneously for the anomalous acceleration and anomalous gravitomagnetic fields around rotating superconductors measured by Tajmar et al. and for the anomalous Cooper pair mass in superconductive Niobium, measured by Cabrera and Tate. It is argued that these three different physical effects are ultimately different experimental manifestations of the simultaneous spontaneous breaking of gauge invariance, and of the principle of general covariance in superconductive materials.

1 Introduction

The existence of dark energy in the universe, as indicated by numerous astrophysical observations, represents one of the most challenging problems

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in theoretical physics at present [1, 2, 3, 4]. A great variety of different models exist for dark energy but none of these models can be regarded as being entirely convincing so far. The cosmological constant problem (i.e. the smallness of the cosmologically observed vacuum energy density) remains an unsolved problem. It is likely that the solution of this problem requires new, so far unknown, physics.

While it is clear that dark energy has measurable effects on cosmological scales (such as the accelerated expansion of the universe as seen from supernovae observations), it is much less clear what the effects of dark energy could be on smaller scales. These effects, if any, depend very much on the model considered. For example, if dark energy is due to the existence of compactified dimensions with a diameter of the order of the micron scale, then this would lead to modifications of the gravitational interaction potential on these scales. This can be tested in laboratory precision experiments. Tests by Adelberger et al. [5] proved negative so far down to a scale of about 50 microns.

Other more recent models of dark energy, such as the electromagnetic dark energy model of Beck and Mackey [6], also produce potentially measurable effects at laboratory scales, which are, however, restricted to the interior of superconductors. In [6] a Ginzburg-Landau theory is constructed that generates a cutoff for the gravitational activity of vacuum fluctuations. Generally it is assumed in this model that vacuum fluctuations of any particle can exist in two different phases: A gravitational active one (contributing to the cosmological constant Λ) and a gravitationally inactive one (not contributing to Λ). The model exhibits a phase transition at a critical frequency which makes the dark energy density in the universe small and finite. The above approach has many analogies with the physics of superconductors, and in particular it allows for a possible interaction between dark energy and Cooper pairs. A suppression of the cosmological constant due to the formation of quantum condensates has also been discussed in [7].

In models of dark energy like the above one it is the *new physics underlying the cutoff* that can potentially lead to measurable effects in the laboratory (see also [8] for related work). In Beck and Mackey's model [6] dark energy couples to superconducting matter only (and not to matter in the normal state). This is theoretically consistent: If we assume that dark energy can interact with superconducting matter only, we do not get any contradiction from cosmological observations, since almost all of the matter in the universe is not in a superconducting state. Given the above assumption of a possible interaction between dark energy and superconducting matter one can then constrain the interaction strength by making precision measurements with superconducting devices.

In this paper we look at recent experiments that were performed with rotating superconductors. There are three observed anomalies that cannot be explained with conventional theories. One dates back already nearly 20 years: Tate et al. [9] made precision measurements of the London moment in rotating superconductors. The London moment is a magnetic field generated inside a superconductor once it is set into rotation. They measured a London moment slightly too large as compared to the theoretical expectations. This anomalous London moment, well established within the experimental precision, has remained unexplained for the past 20 years. More recently, Tajmar et al. [10] investigated rotating superconductors using isolated laser-gyroscopes positioned outside the superconductor. The gyroscopes yield small signals proportional to the rotation frequency that again cannot be explained by conventional theories. These signals may, however, be interpreted in terms of a gravitomagnetic field whose strength is much larger than theoretically expected from ordinary gravity. Finally, Tajmar et al. [11] also measured anomalous induced acceleration signals in isolated accelerometers close to a rotating superconductor, which occur if the rotation frequency of the superconductor is rapidly changed. Again this induced acceleration signal is much stronger than theoretically expected from normal gravity. All three effects are specific to superconducting matter only, they vanish as soon as the temperature exceeds the critical temperature. Presently it seems that none of the above measured effects can be understood in terms of conventional superconductor physics.

We will show that all three effects can be quantitatively understood from a possible interaction between Cooper pairs and dark energy as described in the model of Beck and Mackey [6]. The key for a quantitative understanding of the experimental observation is the fact that photons formally obtain mass in a superconductor, due to a finite London penetration depth. In particular, gravitationally active photons whose vacuum fluctuations underly dark energy in the model [6] obtain a mass as well and lead effectively to a strong enhancement of gravitoelectromagnetic effects, which can be experimentally measured. Ultimately, our theoretical interpretation is that not only gauge invariance but also general covariance is spontaneously broken in the superconducting material, the latter one being related to an interaction of dark energy with Cooper pairs.

This paper is organized as follows: In sections 2-3 we briefly summarize the experimentally observed effects in rotating superconductors that could possibly be linked to dark energy. In sections 4-6 we develop the tools for our theoretical approach, based on the Einstein-Maxwell-Proca equations describing gravitomagnetic fields and the electromagnetic dark energy model of Beck and Mackey. Finally, in sections 7-9 we show that our theory can

explain the experimentally observed effects in a quantitatively correct way.

2 Tajmar's experiments

Tajmar et al. have established a research programme at Austrian Research Centers GmbH -ARC with the objective of peering into new possible gravitational properties of superconductive materials. Two different types of experiments have been carried out:

1. The first category provides measurements of azimuthal accelerations with accelerometers located inside the equatorial plane of the central hole of different types of superconductive rings, which are angularly accelerated. The experimental arrangement was designed to ensure a minimal thermal and mechanical coupling between the accelerometers and the ring's motion. The accelerometers were isolated as much as possible from the whole experimental setup inside a sealed vacuum chamber [10, 11].
2. In the second category, angular velocities were measured with laser gyroscopes located above different types of uniformly rotating superconductive rings (with respective equatorial planes parallel to each other). Like in point 1, the overall experimental setup and the measurement methods ensured that the laser-gyros were maximally decoupled from undesired mechanical torques in general, and from the ring's motion in particular [10].

Based on the known laws of physics, and taking into account all known physical effects in the experimental setup described above in (1) and (2), neither the accelerometers nor the laser-gyros should indicate any significant signal above the noise level. This is not what Tajmar's experiments demonstrated. Rather, a clear azimuthal acceleration, which could be associated with an anomalous gravitational field, directly proportional to the superconductive ring angular acceleration, and an angular velocity orthogonal to the ring's equatorial plane, which could be associated with an anomalous gravitomagnetic field, have been measured in type-(1) and type-(2) experiments respectively [10, 11].

2.1 Acceleration fields around angularly accelerated rotating superconductors

In Tajmar's experiment, the superconducting ring has a radius of $R = 0.07m$ and the accelerometers are positioned inside the central hole of the ring at

distance $r = 0.036m$. In the case of Niobium rings, the measured coupling between the applied angular acceleration $\dot{\omega}$ to the superconducting ring (measured in rad/s^2) and the measured induced azimuthal acceleration g inside the central hole of the ring has the following value and associated error (see table II in [11]):

$$\frac{g}{\dot{\omega}} = -(9.46 \pm 0.28) \times 10^{-7} [m.Rad^{-1}] \quad (1)$$

If the angular frequency is measured in units of $[s^{-1}]$, this result is equivalent to

$$\frac{g}{\dot{\omega}} = -(1.51 \pm 0.04) \times 10^{-7} [m]. \quad (2)$$

Like the gravitational field produced by the mass of a physical body, the measured acceleration, g , does not depend on the accelerometer's mass or chemical composition detecting it. The effect is only seen if the ring is in the superconductive state, i.e. below its respective critical temperature T_c , and if the ring is accelerated. The coupling as given by Eq.(2) depends on the ring's material type, and disappears, within the instrumentation resolution capability, for High- T_c superconductors.

The above value of the coupling is based on single-sensor measurements and the evaluation of maximum acceleration peaks [11]. If multiple sensors at different positions are used (in a so-called curl configuration) and if an average signal analysis is performed, then smaller coupling constants are obtained [10]. However, for the physical interpretation we are going to provide in section 7 (in terms of a particle emission event) the maximum peak analysis with a single sensor is the most appropriate one.

Acceleration peaks were also observed when the superconductor passed through its critical temperature while it was rotating at constant angular velocity. These signals had opposite signs for the transition from the normal to the superconductive state and vice versa.

What could be the origin of the measured anomalous acceleration inside the central region of an angularly accelerated Niobium superconductive ring? What could account for the measured coupling, Eq.(2)? Before we answer these questions let us first investigate the anomalous gravitomagnetic properties of uniformly rotating superconductors.

2.2 Gravitomagnetic fields around uniformly rotating superconductors

In the limit of small field strengths and for nonrelativistic movements, the Einstein equations yield a set of Maxwell-like equations which describe the

so-called gravitomagnetic fields [12]. For normal matter these gravitational fields are much weaker than electromagnetic fields. However, in coherent quantum systems the experiments seem to provide evidence for much stronger than expected gravitomagnetic fields [13].

The most recent experiments of Tajmar et al. use laser-gyroscopes to detect gravitomagnetic fields. A laser-gyroscope is an interferometer, measuring the phase difference between two beams of coherent electromagnetic waves with equal frequency, ν_0 , propagating in opposite directions along a closed optical fiber. When the fiber rotates with angular velocity Ω , a phase difference, $\delta\phi$, is measured. This is the so-called Sagnac effect:

$$\delta\phi = 4\nu_0 \frac{S\Omega}{c^2} \quad (3)$$

Here S is the component of the optical fiber cross section parallel to the rotation plane.

The same effect, a phase difference, can be caused by a gravitomagnetic flux crossing a laser gyroscope at rest, since a gravitomagnetic field \vec{B}_g originates from a vector potential \vec{A}_g :

$$\vec{B}_g = \nabla \times \vec{A}_g \quad (4)$$

In that case the phase difference is directly proportional to the intensity of the gravitomagnetic field:

$$\delta\phi = 4\nu_0 \frac{SB_g}{c^2} \quad (5)$$

Here S is the component of the optical fiber cross section orthogonal to the gravitomagnetic field. Such phase difference are measured by Tajmar et al. [10] when a Niobium superconductive ring is rotating with constant angular velocity ω , with an isolated laser-gyroscope positioned nearby. The measured coupling χ' between the gravitomagnetic field and the angular velocity of the superconductive Niobium ring is of the order

$$\chi' = \frac{B_g}{\omega} \sim 10^{-8}. \quad (6)$$

These gyroscope measurements are not yet finalised and conclusions are still premature. More precise data are expected to become available soon [14].

3 Cabrera and Tate's measurements

Superconductors at rest expell any magnetic field. But when they exhibit rotational motion, a magnetic field \vec{B} is generated within the superconductor,

the so-called London moment:

$$\vec{B} = -2\frac{m}{e}\vec{\omega} \quad (7)$$

Here m and e are the mass and charge of the Cooper pair. This effect is not accounted for by classical electrodynamics, it can only be properly explained in the framework of quantum field theory[15]. It consists in the spontaneous generation of a magnetic field by setting a superconductor into rotation in an environment initially (before the rotation starts) entirely free from any electromagnetic fields.

By measuring the \vec{B} field very precisely, one can conclude on the Cooper pair mass m using Eq.(7). The experimental technique for this is based on magnetic flux quantization: In 1989 Cabrera and Tate [9], through the measurement of the London moment magnetic trapped flux, reported an anomalous Cooper pair mass excess in thin rotating Niobium superconductive rings:

$$\Delta m = m^* - m = 94.147240(21)eV \quad (8)$$

Here $m^* = 1.000084(21) \times 2m_e = 1.023426(21)MeV$ is the experimentally measured Cooper pair mass (with an accuracy of 21 ppm), and $m = 0.999992 \times 2m_e = 1.002331MeV$ is the theoretically expected Cooper pair mass including relativistic corrections. The above Cooper pair mass excess (or, equivalently, the slightly larger than expected measured magnetic field) has not been explained until now.

Motivated by the absence of any apparent solution of this disagreement in the existing literature, one of us (CDM) formulated the conjecture that an additional gravitomagnetic term must be added to the Cooper pairs' canonical momentum:

$$\vec{\pi} = m\vec{v} + e\vec{A} + m\vec{A}_g \quad (9)$$

The gravitomagnetic field strength $B_g = |\vec{B}_g| = |\nabla \times \vec{A}_g|$ required to account for the reported mass excess is 31 orders of magnitude larger than any gravitomagnetic field predicted by general relativity, based on the mass currents in the rotating ring[16, 17]. It can be expressed in terms of Eq.(8) as

$$B_g = \frac{\Delta m}{m}2\omega = 1.84 \times 10^{-4}\omega \quad (10)$$

Here $\vec{\omega}$ is the superconductor's angular velocity. The physical interpretation of Eq.(10) is ambiguous: Either it can be understood as the gravitomagnetic London-type moment in rotating superconductors, or it can be associated with a deviation from the gravitomagnetic Larmor theorem [18], $B_g = 2\omega$,

which would be revealed by anomalous Coriolis forces inside rotating superconductive cavities. These ambiguities will be further discussed and elucidated below. The most important question at this point is why the field B_g is so much larger than expected, and why it is only observed in the superconducting state. In the following we will relate this to an effect produced by dark energy.

4 Massive photons in superconductors

The properties of superconductors (zero resistivity, Meissner effect, London moment, flux quantization, Josephson effect, ...) can be understood from the spontaneous breaking of electromagnetic gauge invariance when the material is in the superconductive phase[15, 19]. In quantum field theory, this symmetry breaking leads to massive photons via the Higgs mechanism. In this case the Maxwell equations transform to the so-called Maxwell-Proca equations, which are given by

$$\nabla \vec{E} = \frac{\rho_e}{\epsilon_0} - \frac{1}{\lambda_\gamma^2} \phi \quad (11)$$

$$\nabla \vec{B} = 0 \quad (12)$$

$$\nabla \times \vec{E} = -\dot{\vec{B}} \quad (13)$$

$$\nabla \times \vec{B} = \mu_0 \rho_e \vec{v} + \frac{1}{c^2} \dot{\vec{E}} - \frac{1}{\lambda_\gamma^2} \vec{A}. \quad (14)$$

Here \vec{E} is the electric field, \vec{B} is the magnetic field, ϵ_0 is the vacuum electric permittivity, $\mu_0 = 1/\epsilon_0 c^2$ is the vacuum magnetic permeability, ϕ is a scalar electric potential, \vec{A} is a magnetic vector potential, ρ_e is the Cooper pair condensate charge density, \vec{v} is the Cooper pair velocity, and $\lambda_\gamma = \hbar/m_\gamma c$ is the photon's Compton wavelength, which is equal to the London penetration depth $\lambda_L = \sqrt{\frac{m}{\mu_0 e \rho_e}}$.

Taking the curl of Eq. (14) and neglecting the term coming from the displacement current, we get the following equation for the magnetic field:

$$\nabla^2 \vec{B} + \frac{1}{\lambda_\gamma^2} \vec{B} = \frac{1}{\lambda_L^2} \frac{m}{e} 2\vec{\omega}. \quad (15)$$

Solving Eq. (15) for the 1-dimensional case, we obtain a magnetic field with a term that decays exponentially and another one that is proportional to the rotation frequency ω . These are respectively the Meissner effect and the London moment:

$$B = B_0 e^{-x/\lambda_\gamma} + 2\omega \frac{m}{e} \left(\frac{\lambda_\gamma}{\lambda_L} \right)^2 \quad (16)$$

Following Becker et al. [20] and London [21], the London moment is developed by a net current that is lagging behind the positively charged ion lattice. The Cooper pair current density shows in opposite direction than the angular velocity of the superconducting bulk. This is important as the London moment in all measurements, due to the negative charge of the Cooper pair, shows in the same direction as the angular velocity. Having

$$\lambda_\gamma = \lambda_L \quad (17)$$

we finally obtain the familiar expression

$$B = B_0 e^{-x/\lambda_L} - 2\omega \frac{m}{e}. \quad (18)$$

The experiments of Tajmar et al. have been operating at temperatures of the order of $7K$. At this temperature, the London penetration depth for Niobium is $\lambda_L = \lambda_L(0)/\sqrt{1 - (T/T_c)^4} = 47.6nm$ (assuming a critical temperature $T_c = 9.25K$, and a London penetration depth at $T = 0$ of $\lambda_L(0) = 39nm$ [22]). Substituting this value into Eq. (17), we deduce a typical value of the photon mass in Niobium

$$m_\gamma = 4.2eV. \quad (19)$$

5 Gravitational Maxwell-Proca equations

In analogy with the electromagnetic fields produced by a Cooper pair condensate, which are described by the set of Maxwell-Proca equations Eq.(11)-Eq.(14), we may write down analogous equations for gravity in the weak field approximation. These generate gravitoelectromagnetic fields according to a set of Einstein-Maxwell-Proca equations, with a massive graviton [23]:

$$\nabla \vec{g} = -\frac{\rho_m^*}{\epsilon_{0g}} - \frac{1}{\lambda_g^2} \phi_g \quad (20)$$

$$\nabla \vec{B}_g = 0 \quad (21)$$

$$\nabla \times \vec{g} = -\dot{\vec{B}}_g \quad (22)$$

$$\nabla \times \vec{B}_g = -\mu_{0g} \rho_m^* \vec{v} + \frac{1}{c^2} \dot{\vec{g}} - \frac{1}{\lambda_g^2} \vec{A}_g \quad (23)$$

Here \vec{g} is the gravitational field, \vec{B}_g is the gravitomagnetic field, $\epsilon_{0g} = 1/4\pi G$ is the vacuum gravitational permittivity, $\mu_{0g} = 4\pi G/c^2$ is the vacuum gravitomagnetic permeability, ϕ_g is the scalar gravitational potential, \vec{A}_g is the gravitomagnetic vector potential, $\rho_m^* = \rho^*/c^2$ is the mass density of the

gravitational condensate (ρ^* is the corresponding energy density), \vec{v} is the velocity of the gravitational analogue of Cooper pairs, and $\lambda_g = \hbar/m_g c$ is the Compton wavelength of the graviton. At the moment, we will leave open the physical interpretation of the gravitational quantum condensate, we will come back to this in the next section. There are several theoretical approaches for the inclusion of a massive graviton in general relativity. Here we adopt the proposal from Deser and Waldron [24], to link the cosmological constant Λ and the graviton mass, m_g , in a partially massless spin 2 theory in a de-Sitter background ($\Lambda > 0$), which describes 4 propagating degrees of freedom for the graviton corresponding to helicities $(\pm 2, \pm 1)$ (helicity 0 being unphysical). In this approach one has

$$\frac{1}{\lambda_g^2} = \left(\frac{m_g c}{\hbar}\right)^2 = \frac{2}{3}\Lambda. \quad (24)$$

At this particular (critical) value a novel local scalar gauge invariance appears that is responsible for the elimination of the helicity 0 excitation. In this theory in the limit of a massless field, $m_g = 0$, the classical two degrees of freedom ± 2 are recovered, and in the interval $0 < m_g^2 < \frac{2\hbar^2\Lambda}{3c^2}$ the theory is unstable. Note that the above graviton mass is tiny: λ_g/c is of the same order of magnitude as the current age of the universe, hence we should not expect any significant modification with respect to the presently experimentally verified laws of gravity from such a tiny postulated graviton mass.

In the 1-dimensional case we obtain the solution of Eq. (23) as

$$B_g = B_{g0} e^{-x/\lambda_g} + 2\omega \left(\frac{\lambda_g}{\lambda_L}\right)^2 \quad (25)$$

where

$$\lambda_L = \frac{1}{\sqrt{\mu_{0g}\rho_m^*}} \quad (26)$$

is the gravitational analogue of the London penetration depth. In Eq. (25) the x -dependence of the first term (the gravitational Meissner effect) can be neglected due to the huge λ_g assumed. The term of interest is the second term, the gravitomagnetic London-type moment, which can potentially lead to measurable effects, depending on what is assumed for ρ^* . The basic idea in the following is to associate ρ^* with dark energy.

6 Dark energy in a superconductor

So far the Einstein-Maxwell Proca equations considered in the previous section were not coupled to the electrodynamics of Cooper pairs. We now consider possible interactions, based on the model of dark energy as proposed by Beck and Mackey in [6].

A non-vanishing cosmological constant (CC) Λ can be interpreted in terms of a non-vanishing vacuum energy density

$$\rho_{vac} = \frac{c^4}{8\pi G}\Lambda, \quad (27)$$

which corresponds to dark energy with equation of state $w = -1$. The small astronomically observed value of the CC, $\Lambda = 1.29 \times 10^{-52} [1/m^2]$ [1], and its origin remain a deep mystery. This is often called the CC problem, since with a cutoff at the Planck scale the vacuum energy density expected from quantum field theories should be larger by a factor of the order 10^{120} , in complete contradiction with the observed value. To solve the CC problem, in [6] a model of dark energy was suggested that is based on electromagnetic vacuum fluctuations creating a small amount of vacuum energy density. One assumes that photons (or any other bosons), with zeropoint energy $\epsilon = \frac{1}{2}h\nu$, can exist in two different phases: A gravitationally active phase where the zeropoint fluctuations contribute to the cosmological constant Λ , and a gravitationally inactive phase where they do not contribute to Λ [6, 25, 26, 27]. This is described in [6] by a Ginzburg-Landau type of theory. As shown in [6], this type of model of dark energy can lead to measurable effects in superconductors, via a possible interaction with the Cooper pairs in the superconductor.

Here we introduce the following additional hypotheses with respect to the original Beck and Mackey model, which, as we shall see later, are consistent with the experimental observations in rotating superconductors:

1. Like normal photons (with energy $\epsilon = h\nu$), the gravitationally active photons (with zeropoint energy $\epsilon = \frac{1}{2}h\nu$) acquire mass in a superconductor due to the spontaneous breaking of gauge invariance. In the following we call these spin 1 bosons *graviphotons*. In Niobium their mass is approximately $\sim 4eV$, as we saw previously in Eq.(19).
2. The transition between the two graviphoton's phases (gravitationally active, versus gravitationally inactive) occurs at the critical temperature T_c of the superconductor, which defines a cutoff frequency ν_c of zeropoint fluctuations specific to each superconductive material: $h\nu_c \sim kT_c$.

3. Graviphotons can form weakly bounded states with Cooper pairs, increasing their mass slightly from m to \tilde{m} . The binding energy is $\epsilon_c = \mu c^2$:

$$\tilde{m} = m + m_\gamma - \mu \quad (28)$$

4. Since the graviphotons are bounded to the Cooper pairs, their zero-point energies form a condensate capable of the gravitoelectrodynamic properties of superconductive cavities.

Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff ν_c of vacuum fluctuations:

$$\rho^* = \frac{1}{2} \frac{\pi \hbar}{c^3} \nu_c^4 \quad (29)$$

In vacuum one may put $\rho^* = \rho_{vac}$, from which the cosmological cutoff frequency ν_{cc} is estimated as

$$\nu_{cc} \simeq 2.01 THz \quad (30)$$

The corresponding "cosmological" quantum of energy is:

$$\epsilon_{cc} = h\nu_{cc} = 8.32 meV \quad (31)$$

In the interior of superconductors, according to assumption 2., the effective cutoff frequency can be different. This is due to interaction effects between the two Ginzburg-Landau potentials (that of the superconducting electrons and that of the dark energy model) [6]. The effect can be seen in analogy to polarization effects of ordinary electromagnetic fields in matter: In matter the electric field energy density is different as compared to the vacuum. Similarly, in superconductors the effective dark energy density (represented by gravitationally active zeropoint fluctuations) can be different as compared to the vacuum. Our model allows for the gravitational analogue of polarization.

An experimental effort is currently taking place at University College London and the University of Cambridge to measure the cosmological cutoff frequency through the measurement of the spectral density of the noise current in resistively shunted Josephson junctions, extending earlier measurements of Koch et al. [28].

In [6] the formal attribution of a temperature T to the graviphotons is done by comparing their zeropoint energy with the energy of ordinary photons in a bath at temperature T :

$$\frac{1}{2} h\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} \quad (32)$$

This condition is equivalent to

$$h\nu = \ln 3kT \quad (33)$$

Substituting the critical transition temperature T_c specific to a given superconductive material into Eq.(33), we can calculate the critical frequency characteristic for this material:

$$\nu_c = \ln 3 \frac{kT_c}{h} \quad (34)$$

For example, for Niobium with $T_c = 9.25\text{K}$ we get $\nu_c = 0.212 \text{ THz}$. If we use the cosmological cutoff frequency in Eq.(34) we find the cosmological critical temperature T_{cc} :

$$T_{cc} = 87.49\text{K} \quad (35)$$

This temperature is characteristic of the BSCCO High- T_c superconductor.

7 Graviphotonic effect in accelerated superconductors

Let us now come to our physical explanation of the observed experimental effects, using the dark energy model of the previous section. In Tajmar's type-1 experiments, a strong angular acceleration applied to the superconductive ring can break the bound between a Cooper pair and its associated graviphoton. In that process the Cooper pair loses the mass $m_\gamma \sim 4\text{eV}$, which by reaction effectively produces an acceleration, on the Cooper pairs, in the opposite direction of the applied acceleration. This generates a macroscopic, measurable acceleration, since a coherent patch of a large number of graviphotons are simultaneously expelled out of the superconductor and absorbed by the vacuum. Our hypothesis is that this produces the measured induced acceleration in Tajmar's experiment, as can be deduced by applying Newton's law of action-reaction to the system formed by a Cooper pair and the graviphoton:

$$-f_{Graviphoton} = -m_\gamma \dot{\omega} R = mg = f_{CooperPair} \quad (36)$$

Here $R \sim 0.07[m]$ is the superconductive Niobium ring radius (in Tajmar's experiment), $f_{Graviphoton}$ is the force applied on the massive graviphotons due to the angular acceleration communicated to the superconductive ring, and $f_{CooperPair}$ is the corresponding reaction force experienced by the Cooper pair,

$\dot{\omega}$ is the superconductor's angular acceleration measured in s^{-2} , and g is the produced acceleration at distance R . From Eq.(36), we find the coupling

$$\frac{g}{\dot{\omega}} = -\frac{m_\gamma}{m}R. \quad (37)$$

Substituting into Eq.(37) the value of the graviphoton mass for Niobium found in Eq.(19), we obtain at distance $R = 0.07m$ the value $g/\dot{\omega} \approx -2.9 \times 10^{-7}[m]$. But the actual acceleration measurements are done with accelerometers that are positioned at the smaller distance $r = 0.036m$, where according to Eq. (22) we expect to see an induced field that is smaller by a factor $r/R = 0.51$. We thus obtain at $r = 0.036m$ the theoretical prediction

$$\frac{g}{\dot{\omega}} \approx -1.49 \times 10^{-7}[m]. \quad (38)$$

This is in excellent agreement with the experimentally measured value as given in Eq. (2), $g/\dot{\omega} = -(1.51 \pm 0.04) \times 10^{-7}[m]$.

If the superconductive ring rotates with constant angular velocity and the temperature T is decreased below T_c , then spontaneously Cooper pairs form and these immediately absorb graviphotons out of the vacuum. By this process their mass increases, and an acceleration signal is detected. Similarly, if T is increased above T_c , then the material immediately loses the graviphotons to the vacuum, the Cooper pair mass decreases, and an opposite acceleration signal is measured, as reported in Tajmar's experiment.

We suggest to call this coherent emission of graviphotons by accelerated superconductors the "*graviphotonic effect*".

8 Gravitomagnetic London moment in rotating superconductors

Let us first provide a short derivation of the ordinary electromagnetic London moment—after that we will proceed to the gravitomagnetic London moment. The Cooper pairs in a superconductor can be regarded as a condensate described by a single wave function Ψ :

$$\Psi = \rho_e^{1/2} e^{i\phi} \quad (39)$$

Here ρ_e is equal to the electric charge density of the Cooper pair condensate, and ϕ is the phase of the wave function. The canonical momentum of the Cooper pairs, $\vec{\pi}$, is proportional to the gradient of the phase of the wave function

$$\vec{\pi} \sim \nabla\phi. \quad (40)$$

Explicitly, the canonical momentum contains a mechanical and a magnetic term:

$$\vec{\pi} = m\vec{v} + e\vec{A} \quad (41)$$

Here again $m \approx 2m_e$ is the Cooper pair mass and e the Cooper pair charge (i.e. twice the electron charge), \vec{v} is the Cooper pair's velocity with respect to an inertial reference frame attached to the laboratory, and \vec{A} is the magnetic vector potential. From Eq.(40) we deduce that the curl of the Cooper pairs' canonical momentum is always zero:

$$\nabla \times \vec{\pi} = 0 \quad (42)$$

Taking the curl of Eq.(41) and using the fact that $\vec{v} = \vec{r} \times \vec{\omega}$ and $\vec{B} = \nabla \times \vec{A}$ we obtain the London moment

$$\vec{B} = -2\frac{m}{e}\vec{\omega}. \quad (43)$$

In close analogy to the above derivation, let us now proceed to a gravitomagnetic London moment as produced by dark energy. As seen before, our central hypothesis is that the gravitational quantum condensate is related to dark energy. We may assume that cosmological quanta of energy $h\nu_{cc}$ manifest themselves as massive particles of mass μ in the superconductor:

$$\mu c^2 = h\nu_{cc} \quad (44)$$

These can be regarded as the gravitational analogues of the Cooper pairs in our model (note the similarity with typical neutrino mass scales). We expect them to be strongly correlated to ordinary Cooper pairs and to rotate with the superconductor. In close analogy to eq. (41) (i.e. replacing $m \rightarrow \mu$, $e \rightarrow -m$, $\vec{A} \rightarrow \vec{A}_g$) we now consider the gravitational canonical momentum

$$\vec{\pi}_g = \mu\vec{v} - m\vec{A}_g - m_\gamma\vec{A}_g, \quad (45)$$

where the term $-m\vec{A}_g$ describes the interaction with the ordinary Cooper pairs of mass m . Assuming that the dark energy quanta, the ordinary Cooper pairs and the graviphotons are described by the same macroscopic wave function (i.e. implementing the assumption of phase synchronisation as in [6]) we again obtain $\vec{\pi}_g \sim \nabla\Phi_g$ where Φ_g is the gravitational phase, hence

$$\nabla \times \vec{\pi}_g = 0. \quad (46)$$

By taking the curl in Eq.(45) we thus obtain the gravitomagnetic London moment

$$\vec{B}_g = 2\frac{\mu}{m + m_\gamma}\vec{\omega}. \quad (47)$$

Putting in $m + m_\gamma \approx m \approx 2m_e$ and $\mu c^2 = h\nu_{cc}$ we obtain the numerical value

$$B_g = 1.62 \times 10^{-8} \omega. \quad (48)$$

This theoretical prediction is consistent with Tajmar's type 2 measurements, Eq.(6), where B_g/ω was measured to be of the order 10^{-8} .

9 Non-classical inertia in superconductive cavities

From the Einstein-Maxwell-Proca equations of our electromagnetic model of dark energy with massive bosons we can derive the inertial properties of a superconductive cavity. Taking the gradient of Eq. (20), and the curl of Eq. (23), and solving the resulting differential equations for the 1-dimensional case we find respectively a form of the principle of equivalence and of the gravitomagnetic Larmor Theorem[18] in superconductive cavities:

$$\vec{g} = -\vec{a}\mu_0 g \rho_m^* \lambda_g^2 \quad (49)$$

$$\vec{B}_g = 2\vec{\omega}\mu_0 g \rho_m^* \lambda_g^2 \quad (50)$$

Here \vec{a} is an acceleration communicated to the superconductive cavity, and \vec{g} is an acceleration measured inside the superconductive cavity. For the derivation of Eq. (49) we assumed the case of a homogeneous field \vec{g} and used the formulas $\vec{g} = -\nabla\phi_g$ and $\rho_m^* \vec{a} = -\nabla(\rho_m^* c^2)$. For Eq. (50) we used Becker's argument that the Cooper pairs are lagging behind the lattice so that the current is flowing in the opposite direction of ω . We now express the coupling $\chi = B_g/\omega$ between the gravitomagnetic field and the superconductor's angular velocity directly as a function of the dark energy density ρ^* contained in the superconductor through the substitution of $\rho_m^* = \rho^*/c^2$ in Eq.(50):

$$\chi = \frac{8\pi G}{c^4} \rho^* \lambda_g^2 \quad (51)$$

Substituting Eq.(27) and Eq.(24) into Eq.(51) and rearranging we obtain

$$\chi = \frac{3}{2} \frac{\rho^*}{\rho_{vac}}. \quad (52)$$

Substituting Eq.(27), Eq.(29) and Eq.(34) into Eq.(52) we obtain

$$\chi = \frac{3 \ln^4 3}{4\pi} \frac{k^4 G}{c^7 \hbar^3 \Lambda} T_c^4. \quad (53)$$

Remarkably, this equation connects the five fundamental constants of nature k, G, c, \hbar, Λ with measurable quantities in a superconductor, χ and T_c .

We may define a Planck-Einstein temperature T_{PE} as

$$T_{PE} = \frac{1}{k} \left(\frac{c^7 \hbar^3 \Lambda}{G} \right)^{1/4} = 60.71 K. \quad (54)$$

Eq.(53) can then be written as [29]

$$\chi = \frac{3 \ln^4 3}{4\pi} \left(\frac{T_c}{T_{PE}} \right)^4. \quad (55)$$

Substituting the critical transition temperature of Niobium, $T_c = 9.25 K$, into Eq.(55) we find the following coupling between the gravitomagnetic field and the angular velocity of a rotating superconductive Niobium ring:

$$\chi = 1.87 \times 10^{-4} \quad (56)$$

This coupling describes the effect of gravitationally active zeropoint energy within the superconductor. The above theoretically predicted value is extremely close to the measured value in Cabrera and Tate's experiment, Eq.(10):

$$\chi = 2 \frac{\Delta m}{m} = 1.84 \times 10^{-4} \quad (57)$$

Let us evaluate the theoretically predicted coupling for various types of superconductors, starting with Aluminium and ending with High- T_c superconductors like YBCO:

Superconductive material	$T_c[K]$	χ
<i>Al</i>	1.18	4.96×10^{-8}
<i>In</i>	3.41	3.46×10^{-6}
<i>Sn</i>	3.72	4.90×10^{-6}
<i>Pb</i>	7.2	6.88×10^{-5}
<i>Nb</i>	9.25	1.87×10^{-4}
High- T_c	79.06	1
<i>BSCCO</i>	87.5	1.5
<i>YBCO</i>	94.0	2

Table 1: Predicted coupling χ between the gravitomagnetic field and the angular velocity for different superconductive materials.

We note that for YBCO, with $T_c = 94.0 K$, we recover the classical gravitational Larmor theorem [18]:

$$B_g = 2\omega \quad (58)$$

From Table 1 we conclude that the effective laws of inertia in superconductive cavities deviate from the laws of classical mechanics, recovering however the classical regime in the limit of YBCO cavities.

At this point one remark is at order: Our theoretical derivation presented in this paper strictly speaking holds only for conventional low- T_c superconductors, because we are using simple Ginzburg-Landau models and BCS-type of arguments for both the superconductor and the dark energy model [6]. A high- T_c superconductor, however, is not described by such a simple theory. Whether a high- T_c superconductor can form bounded states with graviphotons, thus leading to our predicted effects, is theoretically unclear. The measurements of Tajmar do not show any signal above the noise level for high- T_c superconductors. Moreover, the London moment measured for high- T_c superconductors seems not to show any anomalous behavior within the experimental precision [30]. It is likely that our theory holds for conventional superconductors only, where the coupling χ is small.

Our approach raises interesting perspectives for future experiments:

1. Measurements of the Coriolis force in rotating low- T_c superconductive cavities should show a deviation from the predictions of classical mechanics.
2. If in Tajmar's type-2 experiment the laser-gyro is mechanically attached to the rotating Niobium superconductive ring we should find the "gravito-angular" coupling, $\chi = 1.84 \times 10^{-4}$, associated with the Cabrera and Tate Cooper pair mass excess. If instead the laser-gyro is mechanically decoupled from the rotating superconductive ring, which correspond to the current configuration, we should observe the gravitomagnetic London moment characterised by a coupling $\chi' = 1.61 \times 10^{-8}$, Eq.(6), consistent with Tajmar's recent experiments.
3. The dependence of the coupling χ on various superconducting materials should be further investigated. High- T_c superconductors may require a different gravitomagnetic theory.

10 Discussion

Let us end this paper with some general remarks. General Relativity is founded on the *Principle of Equivalence*, which rests on the equality between the inertial and the gravitational mass of any physical system. The *Principle of General Covariance* (PGC) is an alternative version of the principle of equivalence[31], which is very appropriate to investigate the field equations

for electromagnetism and gravitation. It states that *a physical equation holds in a general gravitational field if two conditions are met:*

1. The equation holds in the absence of gravitation; that is, it agrees with the laws of special relativity when the metric tensor $g_{\alpha\beta}$ equals the Minkowsky tensor $\eta_{\alpha\beta}$ and when the affine connection $\Gamma_{\beta\gamma}^{\alpha}$ vanishes.
2. The equation is generally covariant; that is, it preserves its form under a general coordinate transformation $x \rightarrow x'$.

Any physical principle such as the PGC, which takes the form of an invariance principle but whose content is actually limited to a restriction on the interaction of one particular field, is called a dynamic symmetry. Local gauge invariance, which governs the electromagnetic interaction, is an important example of a dynamic symmetry. We can actually say that the Principle of General Covariance in general relativity is the analogon of the Principle of Gauge Invariance in electrodynamics. The breaking of gauge invariance leads to superconducting states. The breaking of general covariance leads to non-conservation of energy-momentum (in the covariant sense)[31]. In this context it is interesting to see that our gravitational dark energy quantum condensate is related to zeropoint fluctuations, for which energy is certainly not conserved.

11 Conclusion

In this paper we have investigated in detail the possibility that the dark energy of the universe may interact with Cooper pairs in superconductors, thus leading to effects that can be observed in the laboratory. Whether or not such an interaction is a realistic assumption depends very much on the dark energy model considered. The electromagnetic dark energy model of Beck and Mackey [6], and its further development as presented in the current paper, naturally contains such an interaction.

There are first experimental hints that one might be on the right track with these types of theoretical models. The graviphotonic effect, the gravitomagnetic London moment, and non-classical inertia in rotating superconductive cavities are three different experimentally observed effects which can all be explained by the proposed model of dark energy — not only qualitatively but also quantitatively. The model ultimately relies on the spontaneous breaking of gauge invariance and the spontaneous breaking of the principle of general covariance in the interior of superconductors.

The considerations presented in this paper, if confirmed by further independent experiments, would imply that the dark energy of the universe

produces measurable effects not only on cosmological scales but also in the interior and the vicinity of superconductors. This opens up the way for a variety of new possible laboratory experiments testing the nature of dark energy and constraining the interaction strength with Cooper pairs. In our model gravitationally active vacuum fluctuations underlying dark energy lead to a strong enhancement of gravitomagnetic fields, in quantitative agreement with the anomalies seen in the experiments of Tate et al.[9] and Tajmar et al.[10, 11].

12 Acknowledgments

We would like to thank Dr. Martin Tajmar for useful comments on an early version of this paper. C.B. is supported by a Springboard Fellowship of EPSRC.

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